Unit 1

Fraction Concepts

The 24-question Diagnostic Test for Fraction Concepts, in multiple-choice format, consists of four parts: (1) introduction to fractions and mixed numbers; (2) factors, multiples, and primes; (3) equivalence and simplest form; and (4) estimation, comparing/ordering, and division interpretation of fractions. The test allows you to pinpoint specific skills and concepts that require more student work. For information on how to use this test to help identify specific student error patterns, see pages 39 through 41.

Pages 159–169 provide blackline masters for worksheets, activities, and instructional games to support the teaching of fraction concepts. See page 40 for a list of the resources for this unit (including technology resources online).

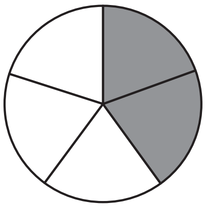
Name ______________________________ Date _______________ Class _______________

Diagnostic Test
Fraction Concepts

Multiple Choice: Circle the correct answer. If your answer is not given, circle "Not here."

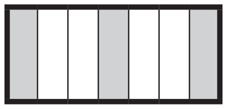
Part 1

1. How much of the circle is shaded?



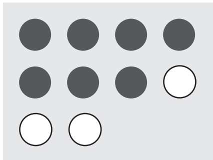
A. $\frac{3}{5}$ **B.** $\frac{2}{3}$ **C.** $\frac{2}{5}$ **D.** $\frac{5}{2}$ **E.** Not here

2. How much of the rectangle is *not* shaded?



A. $\frac{3}{7}$ **B.** $\frac{4}{7}$ **C.** $\frac{4}{3}$ **D.** 4 **E.** Not here

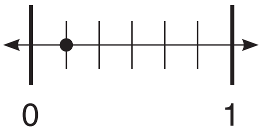
3. What part of the group of circles is shaded?



A. $\frac{7}{10}$ **B.** $\frac{7}{3}$ **C.** $\frac{10}{7}$ **D.** 7 **E.** Not here

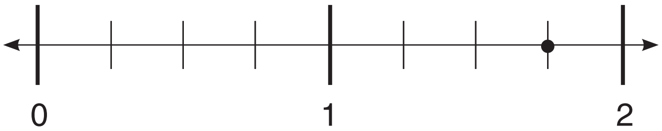
In items 4 and 5, which number names the point shown on the number line?

4.



A. $\frac{1}{5}$ **B.** $\frac{1}{6}$ **C.** $\frac{1}{7}$ **D.** $\frac{2}{7}$ **E.** Not here

5.

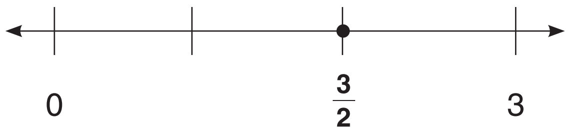


A. $\frac{3}{4}$ **B.** $\frac{7}{8}$ **C.** $1\frac{7}{8}$ **D.** $1\frac{3}{4}$ **E.** Not here

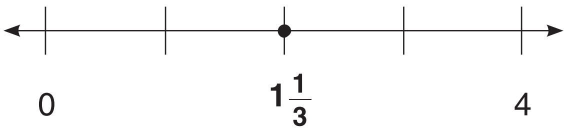
6. On which number line is the point correctly named?

A.

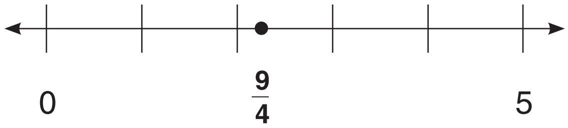
B.



C.



D.



E. Not here

Part 2

7. Which list shows all factors of 18?

A. 2, 3, 6, 9 **B.** 1, 2, 3, 6, 9, 18
C. 1, 2, 6, 9, 18 **D.** 18, 36, 54, . . .
E. Not here

8. Which is *not* a multiple of 35?

A. 5 **B.** 35 **C.** 70 **D.** 105
E. Not here

9. Which is the greatest common factor (GCF) of 24 and 36?

A. 6 **B.** 12 **C.** 18 **D.** 72
E. Not here

10. Which is the least common multiple (LCM) of 20 and 24?

A. 4 **B.** 20 **C.** 120 **D.** 480
E. Not here

11. Which is a prime number?

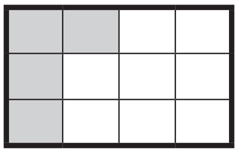
A. 1 **B.** 2 **C.** 25 **D.** 51
E. Not here

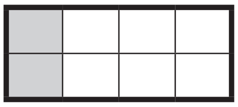
12. Which is the prime factorization of 60?

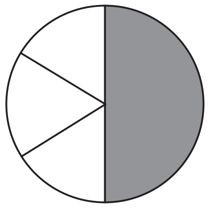
A. 5×12 **B.** $2 \times 2 \times 15$
C. $2 \times 3 \times 5$ **D.** $2 \times 2 \times 3 \times 5$
E. Not here

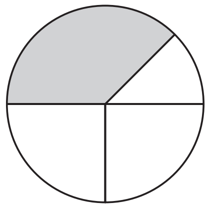
Part 3

13. Which diagram shows $\frac{1}{4}$ shaded?

A.


B.


C.


D.


E. Not here

14. Which is equivalent to $\frac{8}{1}$?

A. 8 **B.** $\frac{1}{8}$ **C.** $\frac{8}{8}$ **D.** 64 **E.** Not here

15. Which is equivalent to $\frac{4}{5}$?

A. $\frac{16}{20}$ **B.** $\frac{9}{10}$ **C.** $\frac{4}{10}$ **D.** $\frac{8}{15}$
E. Not here

16. Which shows $\frac{12}{18}$ in simplest form?

A. $\frac{2}{18}$ **B.** $\frac{6}{9}$ **C.** $\frac{1}{3}$ **D.** $\frac{2}{3}$
E. Not here

17. Which shows an improper fraction for $3\frac{5}{8}$?

A. $\frac{120}{8}$ **B.** $\frac{29}{8}$ **C.** $\frac{24}{8}$ **D.** $\frac{15}{8}$
E. Not here

18. Which shows the mixed number for $\frac{9}{4}$ in simplest form?

A. 2 **B.** $1\frac{5}{4}$ **C.** $4\frac{1}{2}$ **D.** $2\frac{1}{4}$
E. Not here

19. Which tells about how much is shaded?

A. less than $\frac{1}{4}$ **B.** about $\frac{1}{2}$ **C.** about $\frac{2}{3}$ **D.** more than $\frac{3}{4}$

E. Not here

20. Which is correct?

A. $\frac{2}{3} > \frac{3}{4}$ **B.** $2\frac{1}{2} < 1\frac{9}{10}$ **C.** $\frac{12}{18} < \frac{5}{6}$ **D.** $\frac{3}{5} > \frac{5}{8}$

E. Not here

21. Which of these numbers is the greatest, or are they all equal?

$$2\frac{2}{6},\ 2\frac{1}{3},\ \frac{7}{3}$$

A. $2\frac{2}{6}$ **B.** $2\frac{1}{3}$ **C.** $\frac{7}{3}$ **D.** They are all equal.

22. Which is correct?

A. $\frac{2}{5} > \frac{2}{4} > \frac{2}{3}$ **B.** $\frac{11}{12} < \frac{7}{12} < \frac{5}{12}$ **C.** $\frac{1}{2} > \frac{1}{3} > \frac{1}{4}$ **D.** $\frac{6}{10} < \frac{9}{20} < \frac{1}{5}$

E. Not here

23. Which means the same as $\frac{3}{4}$?

A. 3×4 **B.** $4 \div 3$ **C.** $3 \div 4$ **D.** 3.4 **E.** Not here

24. Which shows $\frac{4}{5}$ as a decimal?

A. 0.4 **B.** 0.8 **C.** 1.2 **D.** 1.25 **E.** Not here

ITEM ANALYSIS FOR DIAGNOSTIC TEST
Fraction Concepts

Using the Item Analysis Table

- The correct answer for each item on the Diagnostic Test is indicated by a ✓ in the Item Analysis Table on page 41.
- Each incorrect answer choice is keyed to a specific error pattern and corresponding Intervention Activity found on pages 42 through 85. Because each item on the Diagnostic Test is an item that is analyzed in one of the error patterns, teachers may be able to use the Intervention Activities with identical problems that students may have missed on the test.
- Students should be encouraged to circle "Not here" if their obtained answer is not one of the given answer choices. Although "Not here" is never a correct answer on the Diagnostic Test, the use of this answer choice should aid in the diagnostic process. The intention is that students who do not see their obtained answer among the choices will select "Not here" rather than guess at one of the other choices. This should strengthen the likelihood that students who select an incorrect answer choice actually made the error associated with the error pattern.
- The Item Analysis Table should only be used as a guide. Although many errors are procedural in nature, others may be due to an incorrect recall of facts or to carelessness. A diagnostic test is just one of many tools that should be considered when assessing student work and making prescriptive decisions. Before a teacher may be certain that a student has a misconception about a procedure or concept, further analysis may be needed (see below). This is especially true for students who frequently select "Not here" as an answer choice.
- A set of practice exercises, keyed to each of the four parts of the Diagnostic Test, is provided on pages 86–87. Answers are on page 209. Because the four parts of the set of practice exercises are parallel to the four parts on the Diagnostic Test, the set of practice exercises could be used as a posttest.

Using Teacher-Directed Questioning and Journaling

Discussions and observations should be used to help distinguish misconceptions about concepts and procedures from student carelessness or lack of fact recall. This should be done in a positive manner—with the clear purpose being to "get inside student thinking." The Intervention Activities are replete with teacher-directed questioning, frequently asking students to explain their reasoning. Students should also be asked to write about their thinking as they work through a problem—and, when alternative methods are used, explain why they may prefer one method rather than another.

Additional Resources for Fraction Concepts

Instructional games, worksheets, and activities designed to promote students' understanding of fraction concepts are on pages 159–169 (in blackline master form). Answers are on pages 209–210. Although these materials may be used at any point in the instructional process, specific points of use with teacher suggestions are provided within the Intervention Activities as listed below.

Resource	**Blackline Master(s)**	**Teacher Suggestions**
Number Lines (marked 0 to 1)	160	47–48
Number Lines (marked beyond 1)	161	52–53
Activity: Multiples in *Motion*	162	61–62
Unit Wholes (length model)	163	66–67
Instructional Game: Equivalent Fractions Cover-All	164–165	67–68
Activity: Fraction Card Match	166	75
Benchmark Fractions	167	75
Activity: Paper-Folding to Find Fraction/Decimal Equivalents	168–169	84–85

Technology Resources Online: A list of suggested online interactive software programs (applets) that provide virtual manipulatives, games, activities, and tutorials that you may want to integrate into your lessons is provided on pages 225–227. The resources at each Web site that support this unit are listed under the heading, "Resources for Unit 1: Fraction Concepts."

ITEM ANALYSIS TABLE

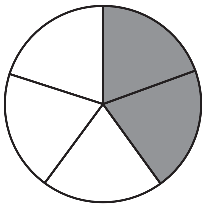
The correct answer for each item on the Diagnostic Test is indicated by a ✓ in this table.

Part	Item	Answer Choices A	B	C	D	Topic	Practice Exercises
	1	Error 1b	Error 1a	✓	Error 1c	Introduction to fractions and mixed numbers	Part 1, p. 86
	2	Error 1b	✓	Error 1a	Error 1d	Introduction to fractions and mixed numbers	Part 1, p. 86
	3	✓	Error 1a	Error 1c	Error 1d	Introduction to fractions and mixed numbers	Part 1, p. 86
	4	Error 2	✓	Error 2	Error 2	Introduction to fractions and mixed numbers	Part 1, p. 86
	5	Error 3b	Error 3a	Error 3a	✓	Introduction to fractions and mixed numbers	Part 1, p. 86
Part 1	6	Error 3a	Error 3c	Error 3b	✓	Introduction to fractions and mixed numbers	Part 1, p. 86
	7	Error 4b	✓	Error 4b	Error 4c	Factors, multiples, and primes	Part 2, pp. 86–87
	8	✓	Error 5	Error 4a	Error 4a	Factors, multiples, and primes	Part 2, pp. 86–87
	9	Error 7c	✓	Error 7a	Error 7b	Factors, multiples, and primes	Part 2, pp. 86–87
	10	Error 8b	Error 8a	✓	Error 8c	Factors, multiples, and primes	Part 2, pp. 86–87
	11	Error 6b	✓	Error 6a	Error 6a	Factors, multiples, and primes	Part 2, pp. 86–87
Part 2	12	Error 6c	Error 6c	Error 6c	✓	Factors, multiples, and primes	Part 2, pp. 86–87
	13	Errors 9a, 11c	✓	Errors 9a, 1e	Errors 9a, 1e	Equivalence and simplest form	Part 3, p. 87
	14	✓	Error 10	Error 10	Error 10	Equivalence and simplest form	Part 3, p. 87
	15	✓	Error 11a	Error 11b	Error 11c	Equivalence and simplest form	Part 3, p. 87
	16	Error 12a	Error 12c	Error 12b	✓	Equivalence and simplest form	Part 3, p. 87
	17	Error 13a	✓	Error 13b	Error 13b	Equivalence and simplest form	Part 3, p. 87
Part 3	18	Error 14a	Error 14b	Error 14c	✓	Equivalence and simplest form	Part 3, p. 87
	19	Error 15	Error 15	Error 15	✓	Estimation, comparing/ordering, and division interpretation	Part 4, p. 87
	20	Error 16a or 16b	Error 16a or 18	✓	Error 16a or 16b	Estimation, comparing/ordering, and division interpretation	Part 4, p. 87
	21	Error 17b	Error 16b	Error 18	✓	Estimation, comparing/ordering, and division interpretation	Part 4, p. 87
	22	Error 17a	Error 17a	✓	Error 15 or 17b	Estimation, comparing/ordering, and division interpretation	Part 4, p. 87
	23	Error 19	Error 19	✓	Error 19	Estimation, comparing/ordering, and division interpretation	Part 4, p. 87
Part 4	24	Error 20	✓	Error 20	Error 20	Estimation, comparing/ordering, and division interpretation	Part 4, p. 87

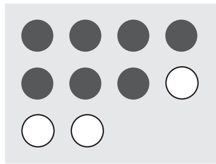
ERROR PATTERNS & INTERVENTION ACTIVITIES
Fraction Concepts

Error Pattern 1

Error Pattern 1a: When writing a fraction to tell how much is shaded, some students compare the number of shaded parts of a figure (or objects in a group) to the number of parts (or objects) that are not shaded.

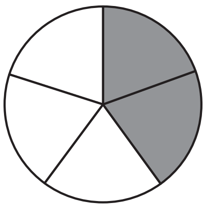


The student responds that $\frac{2}{3}$ of the figure is shaded.

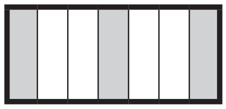


The student responds that $\frac{7}{3}$ of the group of circles is shaded.

Error Pattern 1b: When writing a fraction to tell how much of a figure is shaded, some students compare the parts that are not shaded to the total. When asked to find how much of a figure is not shaded, some students compare the shaded parts to the whole.

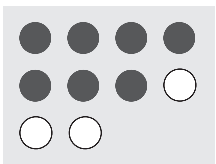


The student responds that $\frac{3}{5}$ of the figure is shaded.

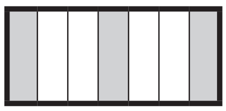


The student responds that $\frac{3}{7}$ of the figure is *not* shaded.

Error Pattern 1c: Some students interchange the numerator with the denominator.

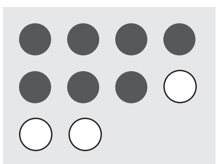


The student responds that $\frac{10}{7}$ of the group of circles is shaded.



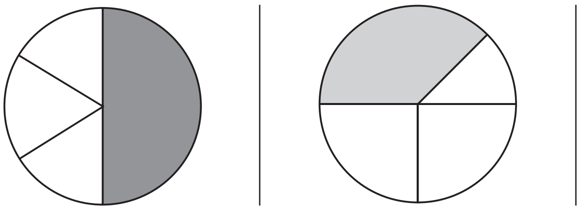
The student responds that $\frac{7}{4}$ of the figure is *not* shaded.

Error Pattern 1d: Some students simply count the number of objects or parts of interest.



The student responds that the part of the group that is shaded is 7.

Error Pattern 1e: Some students do not realize that to write a fraction to show how much of a figure is shaded, the figure must be divided into parts of the same size.



The student responds that each of these figures shows $\frac{1}{4}$ shaded.

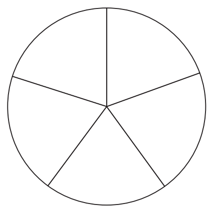
Intervention

Pearn and Stephens (2007), citing the research of others, state that "rational number ideas are sophisticated and different from natural number ideas and that children have to develop the appropriate images, actions, and language to precede the formal work with fractions" (p. 601). The cited research also suggests that a key reason why students have difficulties with fractions is because fractions are the first set of numbers they learn about that are *not* based on a count or a counting algorithm. Rather, fractions show a relationship between a part and a whole. According to Pearn and Stephens (2007), this "required shift in thinking causes difficulty for many students" (p. 601). Thus, it is especially important that early fraction concepts be developed gradually and with meaning. That is the goal of this unit.

Accessing Language (*fracture; fraction*): Mention that if you break a bone, it is called a *fracture*. Usually, such a break divides a whole bone into smaller parts of *different* sizes. Mention that the word *fraction* comes from the Latin *fractus*, meaning "broken" (as in a piece broken off of something). Historically, fractions have often been referred to as "broken numbers."

Display the diagram shown below at right. (A free NCTM applet to create fraction models is available at http://illuminations.nctm.org/ActivityDetail.aspx?ID=44.) **Ask:** "How many parts is the circle divided into? Do the parts look equal in size?" (5; yes.) Ask a volunteer to shade 1 of the 5 equal parts. Explain that a *fraction* can be used to tell what part of the whole you are considering when the parts are *equal in size*. Relate that to "fair shares." Explain that we say, "one-fifth of the circle is shaded," and we write the fraction as $\frac{1}{5}$. When you divide a whole into *equal parts*, each part is a *fraction* of the whole. **Ask:** "How is a fracture different from a fraction?" (Sample: A fracture generally involves parts of different sizes; a fraction involves parts of the same size.)

a fracture



Each part is a *fraction* of the whole because each part is the same size.

Ask: "Suppose the 5 parts of the circle were *not* equal in size. Would it still make sense to say that $\frac{1}{5}$ of the circle is shaded? Explain." (No. Sample: You would *not* be talking about fair shares. In fact, you would not really know what part of the circle was being discussed.)

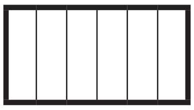
Accessing Language (*numerator; denominator*): Point out that the 5 in $\frac{1}{5}$ is called the *denominator*. Underline the letters *nom* in *denominator*, and mention that in French *nom* means "name." Explain that the denominator of a fraction tells the *name* of the fraction (in this case, fifths). **Ask:** "What does the related word *nominate* mean?" (Sample: *Nominate* means to name someone to run for office.)

Mention that the 1 in $\frac{1}{5}$ is called the *numerator* and that *numerator* comes from the Latin *enumerate,* meaning to count out. So, students can think of the *num* in numerator as meaning "number." Explain that the numerator of a fraction tells the *number* of parts you are talking about (in this case, 1). **Ask:** "What does the related word *numeral* mean?" (Sample: A numeral is a symbol for a number.) According to Rubenstein (2000), we should "teach denominator before numerator. First, we must know a fraction's name, then we learn how many of the parts of the fraction are of interest" (p. 494). For students who have difficulty with the words *numerator* and *denominator*, you may want to use the phrases "top number" and "bottom number."

Accessing Language (*reading and writing fractions*): When reading a fraction, some students say "two-threes" rather than "two thirds." The use of such "whole number" language suggests that the student may be struggling with how the numerator and denominator are related. It is interesting to note that in Korean, Chinese, and Japanese (and other languages), a fraction, say, *two thirds*, is read "of three parts, two." Based on research with first and second graders, Miura and Yamagishi (2002) concluded that "the Korean vocabulary of fractions appeared to influence conceptual understanding and resulted in the children having acquired a rudimentary understanding of fraction concepts prior to formal instruction" (p. 207). Miura and Yamagishi stress the importance of providing "linguistic support" when teaching fraction concepts.

Encourage students to write fractions with a *horizontal bar*, not a slanted bar (as in 3/5). A horizontal bar allows for the use of the phrases "top number" and "bottom number." It also reduces possible confusion when writing mixed numbers ($2\frac{3}{5}$ is clearer than 2 3/5.) It should be noted that students *will* use a slanted bar when they ultimately work with certain calculators and computer spreadsheets. You may want to mention that the fraction bar that separates the numerator from the denominator is called a *vinculum.* It comes from the Latin *vincio*, meaning "to bind."

Part-Whole Model: Display the rectangle shown below. **Ask:** "How many equal parts is the rectangle divided into? What do we call those parts?" (6; sixths.) Explain that the fractional parts are called *sixths* because 6 equal parts are needed to make up the whole.



An effective strategy to determine how many parts of a unit whole should be shaded to represent a given fraction is to count fractional parts. Have students shade 1 part of the above rectangle and say, "One sixth is shaded." Then have students shade another part of the rectangle and say, "Two sixths are shaded." Repeat until all 6 parts are shaded. Students should understand that they are counting sixths—"*one* sixth, *two* sixths, *three* sixths," and so on—akin to how they count objects (*one* apple, *two* apples, and so on).

> "Counting fractional parts to see how multiple parts compare to the whole creates a foundation for the two parts of a fraction."
>
> —Van de Walle (2006, p. 298)

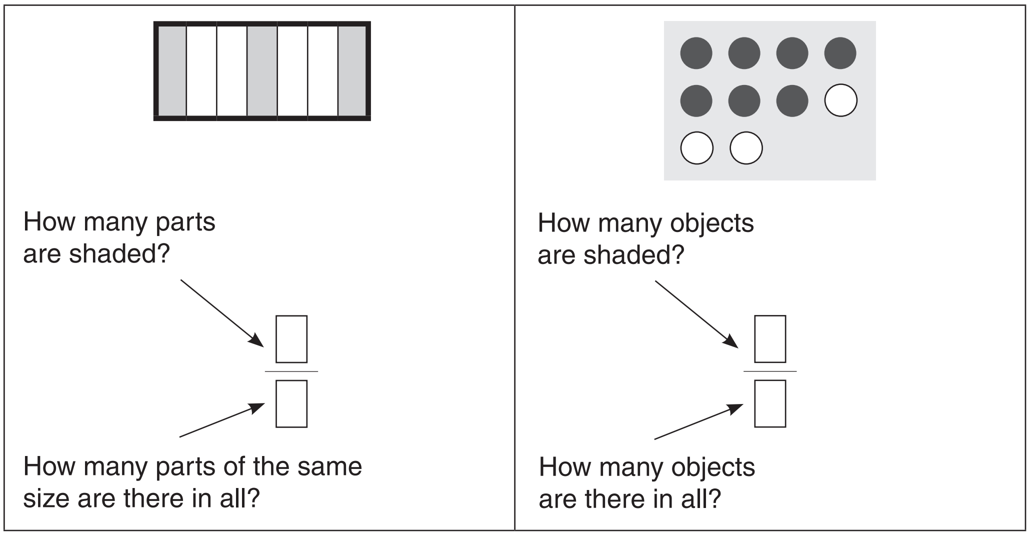
Ask questions such as the following:

- Suppose you shade five sixths and then shade another sixth. How much of the rectangle is shaded? How much of the rectangle is not shaded? $\left(\frac{6}{6}, \text{ or } 1; \frac{0}{6}, \text{ or } 0.\right)$
- Suppose you shade the entire rectangle and then erase the shading on two sixths. How much of the rectangle is shaded? How much is not shaded? $\left(\frac{4}{6}; \frac{2}{6}.\right)$

Counting fractional parts is a strategy that may be used when adding fractions with like denominators. In an Intervention Activity for Error Pattern 2 in Unit 2, students decompose fractions (into unit fractions) and then count fractional parts.

Parts of a Collection Model: Distribute small objects (such as two-color chips) to reinforce student understanding of the "parts of a collection" model of fractions. Emphasize that the entire group of objects is considered to be one whole, and we are looking for what portion of the group is either shaded or not shaded.

For additional practice with the part-whole and parts of a collection models, have students complete exercises prepared as shown on page 46. For the unit wholes, include both rectangles and circles. Include examples where all of the parts or objects are shaded (fractions for 1) and where none are shaded (fractions for 0). Also include questions that ask for students to tell what fraction is *not* shaded.



Accessing Language (*avoiding the phrase "out of"*): Siebert and Gaskin (2006) point out that a common way that some students refer to a fraction, such as $\frac{5}{8}$, is with the phrase "5 out of 8." But according to them, "With an 'out of' image, children see themselves presented with 8 things, then taking 5 from those 8 things. In this image, the numerator and denominator of the faction are merely whole numbers. The 8 things are not thought of as eighths, nor is the 5 conceived of 5 one-eighths" (p. 397). Siebert and Gaskin advocate using the phrase "5 one-eighths," because that language conveys an image of partitioning (the whole is divided into 8 equal parts, each being 1 eighth) and iterating (there are 5 parts under consideration).

Some of the negative implications that could occur if students use the "out of" terminology discussed by Siebert and Gaskin are paraphrased below.

- Improper fractions may become difficult to understand. Does $\frac{4}{3}$ mean you take 4 parts out of 3? Can you take 4 things from 3 things?
- By considering a fraction as two whole numbers, students may think 3 blue chips out of 10 chips is greater than 1 blue chip out of 2 (because 3 chips are more than 1 chip). So why *isn't* $\frac{3}{10} > \frac{1}{2}$?
- In baseball, a player who gets 2 hits "out of" 3 at-bats and then gets 1 hit "out of" 4 at-bats has 3 hits "out of" 7 at-bats. So why *doesn't* $\frac{2}{3} + \frac{1}{4} = \frac{3}{7}$?

Error Pattern 2

When using a number line to name a proper fraction (a fraction whose numerator is less than the denominator), some students count the number of hash marks *between* 0 and 1 to determine the denominator. Others count all of the hash marks—including those for 0 and 1. Some students count the hash mark for 0 when determining the numerator. In general, the student uses a