Chapter 3

Tricks with Dice

Dice are usually associated with games—gambling games as well as board games. Dice can also be the central feature of a magic trick. Each of the magic tricks in this chapter use dice. Most of these tricks work because of the way dice are designed· On any die the sum of the two numbers on a pair of opposite faces is always seven.

3.1 Predicting Numbers on Dice

Mathematical Prerequisites

Computation, expanded notation, use of variables, simplifying polynomials, distributive property

Description

You can accurately tell a volunteer the numbers on the dice he or she has tossed after he or she gives you the final result of a series of simple calculations.

Directions for the First Trick

Turn your back to the volunteer, and give these instructions:

- Toss a pair of dice
- Multiply one of the numbers on the dice by 5
- Add 8 to the product
- Multiply the sum by 2
- Add the number on the other die to the product
- Tell me your final result

You then announce the numbers that were tossed.

Secret of the First Trick

Subtract 16 from the volunteer's final answer. The numbers on the dice are the two digits of the numeral of the resulting number

Analysis of the First Trick

Let x be the first number and y the other number. The series of calculations yields

$$(5x + 8) \cdot 2 + y$$

which simplifies to

$$10x + y + 16.$$

Subtracting 16 leaves

$$10x + y = x(10) + y(1).$$

Thus, the ten's digit and the one's digit are the numbers showing on the dice

Directions for the Second Trick

Turn your back to the volunteer, and give these instructions.

- Toss three dice.
- Multiply the number on one die by 2
- Add 5 to the product.
- Multiply the sum by 5
- Add the number on another die to the product.
- Multiply the sum by 10.
- Add the number on the third die to the product
- Tell me your final result.

You then announce the numbers that were tossed.

Secret of the Second Trick

Subtract 250 from the volunteer's final answer. The numbers on the dice are the three digits of the numeral of the resulting number.

Analysis of the Second Trick

Let x, y, and z be the numbers on the three dice The series of calculations yields

$$[(2x + 5) \quad 5 + y] \quad 10 + z$$

which simplifies to

$$100x + 10y + z + 250$$

Subtracting 250 leaves

$$100x + 10y + z = x(10^2) + y(10) + z(1).$$

Thus, the hundred's digit, ten's digit, and one's digit are the numbers showing on the three dice.

Follow-up Activity

Create different sets of instructions for tossing two dice or three dice Design a set of instructions for tossing four dice

References

Ball, p 12
Gardner, *Mathematics, Magic and Mystery,* pp. 44–45

3.2 Divining the Sum of Three Dice

Mathematical Prerequisites

Computation, use of variables, simplifying polynomials, adding/subtracting polynomials

Description

You are able to tell a volunteer the sum of the numbers he or she has added from certain faces of three dice

Directions for the Trick

Turn your back to the volunteer, and give these instructions:

- Toss the three dice, and add the numbers that show on the top faces
- Choose one of the dice, and add the number on its bottom face to your sum
- Toss this die again, and add the number that turns up to your total

Turn around, and announce the volunteer's total

Secret of the Trick

When you turn around, look quickly at the numbers showing on the three dice Add those numbers, and then add 7 to that sum to get the volunteer's total

Analysis of the Trick

This trick works because on any die the sum of the two numbers on any pair of opposite faces is always 7

Let a, b, and c be the numbers that show after the first toss The volunteer's first sum is $(a + b + c)$. Suppose that the volunteer picks up the die which shows a The number on the bottom face of that die is $(7 - a)$ So, the second sum is

$$(a + b + c) + (7 - a) = b + c + 7$$

Suppose that the number d shows after that particular die is tossed again Then, the final sum is

$$(b + c + d + 7)$$

When the magician turns around, the numbers on the dice that show are b, c, and d. To get the volunteer's total, the magician only needs to add 7 to the sum $(b + c + d)$.

Follow-up Activity

Discuss how to generalize this trick for tossing n dice if $n > 3$.

References

Gardner, *Mathematics, Magic and Mystery*, p. 43
Johnson and Glenn, p 267
Wyler and Ames, p 37

3.3 Prognosticating Products with Dice

Mathematical Prerequisites

Computation, use of variables, simplifying polynomials, multiplying polynomials, distributive property

Description

You can easily predict the sum of four products that a volunteer will compute from the numbers on a pair of dice

Directions for the Trick

Write 49 on a slip of paper. Fold the paper and give it to someone to hold for safekeeping Ask a volunteer to toss a pair of dice and write down the results of the following computations

- Multiply the two top numbers on the dice
- Multiply the two bottom numbers on the dice
- Multiply the top number on one die by the bottom number on the other die
- Multiply the other pair of top and bottom numbers
- Now, add up the four products, and announce the sum

Ask the person holding the folded slip of paper to unfold it and read your prediction.

Secret of the Trick

This trick is self-working The sum of the four products will always be 49. For this reason, the trick should not be repeated

Analysis of the Trick

This trick works because on any die the sum of the two numbers on any pair of opposite faces is always 7

Let a and b be the numbers that show after the dice are tossed. The four products are ab, $(7-a)(7-b)$, $a(7-b)$, and $b(7-a)$ Adding and simplifying these products yields

$$ab + (7-a)(7-b) + a(7-b) + b(7-a)$$

$$= ab + 49 - 7a - 7b + ab + 7a - ab + 7b - ab$$

$$= 49$$

Follow-up Activity

Investigate how to do this trick if three dice are tossed The volunteer will have to compute 12 products Show that the predicted sum of the 12 products must be 147.

Reference Fraser, p. 10

3.4 Prophecy with Colored Dice

Mathematical Prerequisites

Computation, use of variables, adding/subtracting polynomials

Description

You can prophesy the sum that three volunteers will generate from the numbers on three dice

Directions for the Trick

You need three dice of different colors: white, red, and green You also need three slips of paper labeled "white and red," "red and green," and "white and green," a sheet of paper, and a pencil

Write "the total is 21" on a separate slip of paper. Fold the paper, and give it to someone to hold for safekeeping. Then, ask for three volunteers, and number them Volunteer #1, Volunteer #2, and Volunteer #3

Turn your back to the volunteers, and tell them to do the following

- One of you toss the three dice Then, each of you choose one of the three slips of paper
- Volunteer #1, add mentally the numbers on the two dice of the colors on your slip of paper, and write the sum on the sheet of paper.
- Volunteer #2, turn over the two dice of the colors on your slip of paper, add those numbers mentally, and write the sum on the sheet of paper
- Volunteer #3, turn over the two dice of the colors on your slip of paper, add those numbers mentally, and write the sum on the sheet of paper.
- One of you add the three numbers on the sheet of paper, and announce the sum.

Ask the person holding the folded slip of paper to unfold it and read your prediction.

Secret of the Trick

This is another self-working trick The sum of the three numbers will always be 21 For this reason, the trick should not be repeated

Analysis of the Trick

This trick works because on any die the sum of the two numbers on any pair of opposite faces is always 7

Suppose that the colors chosen by the volunteer are

Volunteer #1	white and red
Volunteer #2	red and green
Volunteer #3	white and green ,

and suppose that after the dice are tossed the white die shows x, the red die shows y, and the green die shows z. The sums that the volunteers compute are

	White Die	**Red Die**	**Green Die**	**Volunteer's Sum**
Volunteer #1	x	y	z	$x + y$
Volunteer #2		$7 - y$	$7 - z$	$14 - y - z$
Volunteer #3	$7 - x$		z	$7 - x + z$.

The total of the three sums is

$$(x + y) + (14 - y - z) + (7 - x + z) = 21.$$

Follow-up Activity

Design a similar self-working trick that uses four volunteers and four dice of different colors. What combinations of two colors can you write on the four slips of paper so that the total is always 28?

References Fulves, *Self-Working Table Magic*, pp 31-32