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This middle section contains what we call *families* of problems for groups, and is occasionally interrupted by a couple pages of advice. Look on the next two pages for an expanded listing of the problem families and the advice they surround.

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Introduction

You're holding in your hands a collection of math problems from the EQUALS program at the Lawrence Hall of Science (University of California at Berkeley). The problems span grades 4–12, include a wide range of mathematics topics, and are presented in a special format. They are problems for groups, not individuals, to solve together.

Each problem is in the same format: the information a group will need to know has been put on clue cards. Each member of the group will have a different bit of information, so everyone will have to cooperate to solve the problem. This particular format for group problems—though not the alpha and omega of cooperative tasks—is an especially easy one for teachers to begin with in mathematics.

Cooperation, Equity, and Math Learning

When we introduce problems like these to students, we explain the rules, and focus on the idea that their *groups* will have problems to solve. Then the groups get to work. They get pretty excited. After we've done a few problems, we stop and talk about what it was like to do math problems this way.

Everybody has a part to do is the first thing they say. That's the biggest benefit from an equity point of view. When teachers talk, many students stop paying attention. When students are in a group, they are almost always engaged.

It's easier with many brains is another typical comment. True. Chances are that somebody else sees things we don't.

I saw someone else work the problem a different way is another observation students make. There are many ways to solve a problem; we may gain insight into mathematical models that will help us beyond this one problem.

The group used a lot of math words in the discussion. Using mathematical language is more significant than a simple vocabulary drill. "Talking math" helps us cement our understanding of the mathematics behind the terms.

We knew when we were right. When students are doing the problems, we tend not to tell groups if their answers are right, and we don't answer questions unless everybody in the group agrees they need help. Working in a group fosters independence in learning. Students realize that the teacher is not the sole source of knowledge. Peers can explain things and assess whether they've done it right or not.

Some aspects of cooperative learning benefit the teacher as well:

There are fewer "units" to supervise. Instead of thirty-two students, you will have eight groups. That means you are more free to encourage groups who confront really perplexing problems.

The teacher can spend more time watching the students. As you circulate around the room, you will see the students solving problems. Because they become self-sufficient (or maybe group-sufficient) there will be more time for you to observe; you'll have a better chance of finding out what they're really thinking.

The result of all this is pretty rosy; we can go on:

- Doing group work as well as lecture and seat work appeals to more modalities; if we're interested in equity, we'll want to try to offer as many different ways of learning as possible.
- Problems like these can open the door to including other disciplines in the math classroom, thereby integrating the curriculum. This will also help underserved students by appealing to more interests and showing that math might be important outside of math class.
- Another regular outcome of these problems is success. Groups do *solve* these problems; often problems much more complex than individuals in the group could solve. This success can be a great boost to morale—theirs and yours.
- All students—from the most gifted to the slowest—benefit from learning to explain themselves more clearly and concisely.
- You may observe "weak" students being effective and contributing group members, and "strong" students may discover that they can get in the way of their group's solution.
- Learning to work together is important in itself. Most jobs require it, though it doesn't seem to be taught explicitly anywhere. Nowhere is this more true than in math-related fields.

Cooperation in the math classroom helps us advance several goals as we provide good, equitable mathematics education. We want to put students in genuine problem-solving situations, and to observe students working on problems so we can assess what they really understand. We want problems to be complex but not intractable, and we want students to persist and succeed. We want to help students become autonomous learners, independent of the teacher and free of pat answers.

We hope students will make mathematics their own, talking math and building understanding by solving problems with others. We hope they will be flexible and have access to many strategies, that they will see that mathematics is not an activity undertaken in isolation. Mathematics demands working together and listening as well as raw analytical prowess. All of these goals—every single one—are advanced by students working together effectively in groups.

Assessment

How do you assess cooperative lessons? Assessment will have to change: tests alone won't tell the story. And you must assess yourself and the lesson as well as the students and the groups.

There are many strategies for getting rich information from groups and individuals about what they understand—both about mathematics and about the social aspects of cooperation. In a section beginning on page 112, we'll talk about classroom observation and student writing as alternatives to traditional testing.