

# **Differentiated Instruction for Mathematics**

Instructions and activities for the diverse classroom

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# Introduction

Mathematics is the key to opportunity. . . . For students, it opens doors to careers. For citizens, it enables informed decisions. For nations, it provides knowledge to compete in a technological economy.

—NATIONAL RESEARCH COUNCIL (1989)

To meet the needs of all students and design programs that are responsive to the intellectual strengths and personal interests of students, we must explore alternatives to traditional mathematics instruction. We need to examine not only what is taught but how it is taught and how students learn.

Carol Ann Tomlinson in *The Differentiated Classroom: Responding to the Needs of All Learners* encourages educators to look at teaching and learning in a new way. Using the phrase “One size doesn’t fit all,” she presents, not a recipe for teaching, but a philosophy of educational beliefs:

- Students must be seen as individuals. While students are assigned grade levels by age, they differ in their readiness to learn, their interests, and their style of learning.
- These differences are significant enough to require teachers to make accommodations and differentiate by content, process, and student products. Curriculum tells us what to teach; differentiation gives us strategies to make teaching more successful.
- Students learn best when connections are made between the curriculum, student interests, and students’ previous learning experiences.
- Students should be given the opportunity to work in flexible groups. Different lessons point toward grouping students in different ways: individually, heterogeneously, homogeneously, in a whole group, by student interests, and so forth.
- There should be on-going assessment—assessment can be used to help plan effective instruction.

To address the diverse ways that students learn and their learning styles, we can look to Howard Gardner’s eight intelligences to provide a framework. Gardner’s theory of multiple intelligences encourages us to scrutinize our attitudes toward mathematical learning so that each student can learn in a more relaxed environment.

Let's explore what multiple intelligences look like in the mathematics classroom.



### **Visual/Spatial**

Perceives the visual world with accuracy; can transform and visualize three dimensions in a two-dimensional space. Encourage this intelligence by using graphs and making sketches, exploring spatial visualization problems, relating patterns in math to visual and color patterns, using mapping activities, and using manipulatives to connect concrete with abstract.



### **Verbal/Linguistic**

Appreciates and understands the structure, meaning, and function of language. These students can communicate effectively in both written and verbal form. Encourage this intelligence by using class to discuss mathematical ideas, using journals to explore mathematical ideas using words, making written and oral presentations, and doing research projects.



### **Logical/Mathematical**

Ability to recognize logical or numerical patterns and observe patterns in symbolic form. Enjoys problems requiring the use of deductive or inductive reasoning and is able to follow a chain of reasoning. Encourage this intelligence by organizing and analyzing data, designing and working with spreadsheets, working on critical-thinking and estimation problems, and helping students make predictions based upon the analysis of numerical data.



### **Musical/Rhythmic**

The ability to produce and/or appreciate rhythm and music. Students may enjoy listening to music, playing an instrument, writing music or lyrics, or moving to the rhythms associated with music. Activities related to this intelligence include using songs to illustrate math skills and/or concepts and connecting rational numbers to musical symbols, frequencies, and other real-world applications.



### **Bodily/Kinesthetic**

The ability to handle one's body with skill and control, such as dancers, sports stars, and craftspeople. Students who excel in this intelligence are often hands-on learners. Activities related to this intelligence include the use of manipulatives, involvement with hands-on activities (weighing, measuring, building), and permitting students to participate in activities that require movement or relate physical movements to mathematical concepts.



### Interpersonal

The ability to pick up on the feelings of others. Students who excel in this intelligence like to communicate, empathize, and socialize. Activities related to this intelligence include using cooperative-learning groups, brainstorming ideas, employing a creative use of grouping (including heterogeneous, homogeneous, self-directed, and so forth), and using long-range group projects.



### Intrapersonal

Understanding and being in touch with one's feelings is at the center of this intelligence. Activities related to this intelligence include encouraging students to be self-reflective and explain their reasoning, using journal questions to support metacognition, and giving students quiet time to work independently.



### Naturalist

Naturalist intelligence deals with sensing patterns in and making connections to elements in nature. These students often like to collect, classify, or read about things from nature—rocks, fossils, butterflies, feathers, shells, and the like. Activities related to this intelligence include classifying objects based upon their commonalities, searching for patterns, and using Venn diagrams to help organize data.

## The Format of the Book

The National Council of Teachers of Mathematics (NCTM) in *Principles and Standards for School Mathematics* (2000) refined the 1989 standards by delineating content and process goals essential for all students, grades K–12. The chapters of this book have been organized around the content and process standards defined by the NCTM—numbers and operations, algebra, geometry and measurement, and data analysis and probability. The activities and projects in each chapter reflect the philosophy of differentiation, provide a math curriculum that is standards-based, and involve students in hands-on, motivating real-world problems. Each chapter ends with a “Brush Up Those Skills” game that supports flexible grouping and employs the skills and concepts introduced in the chapter’s activities. The Appendix contains copies of a blank Teacher’s Page, Task/Audience/Product (TAP) Activities page, and “Brush Up Those Skills” pages to help the teacher design and organize his or her own differentiated mathematics lessons. One of the lessons, “How Long Is Your First Name?” requires students to use one-inch squares, and a “Brush Up Those Skills” activity requires a copy of Pascal’s triangle—so these sheets have been provided, as well. The Teacher’s Pages require additional discussion.

## Using the Teacher’s Pages

Each mathematical experience is preceded by a Teacher’s Page that includes valuable information for managing the lesson. These pages have been designed to merge the NCTM’s mathematics standards and the philosophy of differentiation.

- **Math Topics:** As in most hands-on activities, these mathematical experiences address more than one math skill or topic. In the real world, mathematics is an integrated experience, and skills and concepts interrelate and blend. When using the activities, teachers can use this section to connect the lesson to skills and concepts that are part of their mathematics curriculum.
- **Prior Knowledge Needed:** Differentiating necessitates that the teacher know their students’ prior knowledge. This information can be gained in a variety of ways—through pretesting, observation and questioning, or available data from other sources. To be assured that each student’s experience is meaningful and enriching, it is important to know where to start.

- **Differentiation Strategies**

**Principles:** The three principles of differentiation are respectful tasks, flexible grouping, and ongoing assessment. Applicable principles are discussed in this section.

**Teacher’s Strategies:** Teachers can differentiate content, process, and/or product. Every student should be exposed to a mathematics curriculum that is equitable, essential, and requires higher-level thinking skills. Those differentiation strategies that can be used to accommodate diverse learners can be found in this section.

**According to Students:** Students’ readiness, interests, and learning styles determine the accommodations made in a differentiated classroom. Pretest and interest surveys can be taken at any time to meet student needs. This section suggests the multiple intelligences and learning styles that are highlighted in the activity.

- **Materials Needed**

A comprehensive list of materials and supplies needed for each activity is provided. To help the activity run more smoothly, these should be gathered and made ready prior to the lesson.

- **Teaching Suggestions**

**Engaging the Students:** Suggestions are made to begin the lesson and perk student interest. There may be a song or a poem that will draw student attention. This engaging activity varies with the lesson. Productive questioning is sometimes suggested to focus students on the lesson. Productive questioning include these questions:

- Focus attention (What have you noticed about . . . ? What do you see when you . . . ?)
- Help students count or measure (How many . . . ? How long . . . ? How much . . . ?)
- Comparison questions (What do these have in common with . . . ? How are they different?)

- Problem-posing questions (What problems did you face when . . . ? How did you solve this problem?)
- Reasoning questions (Why do you think . . . ? What is your reason for . . . ? Can you come up with a rule for . . . ?)

These types of questions can also be used during debriefing or as ongoing assessment during observation of students.

**The Exploration:** “Engaging the Students” usually has students discussing and working as a whole group. If students are going to work individually, with a partner or in a group of four, the regrouping will be done at this time. While this section is not a scripted, step-by-step plan, it does give suggestions to help teachers encourage students and make the experience more meaningful. Some lessons do suggest specific questions that will help students focus or develop understanding. But these are merely suggestions and should be used only if appropriate to the needs of the class. Available answers also appear in this section.

**Debriefing:** During this time groups, will come back together to discuss their findings and share their results. Many of the activities ask students to share their data on a group data table for further analysis. Productive questions can be used at this time to help focus students’ attention on the important concepts and skills presented in the activity.

- **Assessment**

Multiple suggestions are made here. They may include the following: (1) completed student products, (2) observation and questioning, (3) tiered or non-tiered journal question(s), and (4) TAP activities.

Many types of activities can be tiered, such as assignments, journal questions, warm-ups, and activities. A few suggestions to help plan tiered activities are as follows:

1. All levels should apply the same skill or concept.
2. All levels should meet student readiness.
3. All levels should challenge students and provide new learning experiences.
4. Higher difficulty levels should be a more faceted problem, have less structure, and require more independence of the students.
5. All levels should build on student knowledge and require higher level thinking skills.

Task/Audience/Product (TAP) activities are adapted from Tomlinson’s RAFT activities. They are used sparingly in assessment suggestions, but a blank form is located in the Appendix section of this book. This form can be used to develop your own assessment activities.

- **Variations for Differentiation (Tiered Activities):** Some of the activities can be tiered or elaborated upon, and suggestions have been made in this section. If the teacher has any activities that can be added to enhance the lesson, this would be a fine place in which to list those for future use.