

Number Theory, Numeration, and Computation

There is nothing so troublesome to mathematical practice . . . than multiplications, divisions, square and cubical extractions of great numbers . . . I began therefore to consider . . . how I might remove those hindrances.

—JOHN NAPIER

Number theory, numeration, and computation remain important components of the current school mathematics curriculum. While it is true that computation and basic number facts have been emphasized to the detriment of other strands of mathematics, we all understand that student proficiency in these areas is essential for students to be successful in understanding math concepts. With guidance and meaningful experiences, students will gain a sense of number, improve their ability to solve problems, and develop useful strategies to estimate reasonableness of answers. The activities and projects in this chapter will encourage the development of number and operation sense in students.

“The Bowling Game” (page 3) gives students the opportunity to practice order of operations and computational skills in an interesting way. The numbers used in the game are determined by chance according to the roll of one die. Individual creativity and imagination are rewarded in points gained. The lesson can be augmented by using a little ditty, “O³—Order of Operations Song” (page 6). This makes learning the rules even more fun and draws attention to the musical/rhythmic intelligence!

“All About the Moon” (page 8) demonstrates the connections between math and science while presenting computation, estimation, conversions, and open-ended problem solving as captivating activities. Interesting moon facts are explained in a way that makes sense to students, and very large numbers become more understandable through hands-on activities. By encouraging students to work collaboratively, a variety of differentiation strategies can be utilized.

When patterns found in numbers are translated into fascinating visual patterns, students can investigate the relationship between art and mathematics; skills and concepts are learned using a variety of learning styles and intelligences. In “The Pattern Tells It All” (page 12), students learn how prime and composite numbers form patterns in number grids that are visually different. By writing about their patterns, students verbalize the depth of their understanding.

When students can see connections between school math and real math, they are motivated to learn important concepts and skills. “Baking Blueberry Muffins” (page 18) shows students mathematics applications that utilize ratio and proportion, percent of profit, and everyday conversions. While students work together on this activity, they make use of many of the strategies of differentiated instruction.

By using the rules of divisibility and some simple computation, students can discover all of the prime numbers less than 500 using “Eratosthenes and the 500 Chart” (page 21). “The Divisibility Ditty” (page 115) reminds students of the divisibility rules while encouraging students to use a variety of intelligences to learn important mathematics concepts.

Finally, “Diagramming Divisibility” introduces Venn diagrams—using logic to reinforce divisibility rules in yet another way. “The Divisibility Ditty” can be used again to review these important rules.

Patterns, Functions, and Algebra

In mathematics he was greater
Than Tycho Brahe, or Erra Pater:
For he, by geometric scale,
Could take the size of pots of ale;
Resolve, by sines and tangents straight,
If bread or butter wanted weight
And wisely tell what hour o' th' day
The clock does strike by algebra.

—SAMUEL BUTLER (1612–1680)

When students study algebra the most common question usually is, “When are we ever going to have to use this?” Rather than present algebra as a set of rules and procedures, the lessons in this chapter relate algebra to real-world phenomena and motivating activities.

We start off with a contest—which pair of students can land their parachutes the closest to each other? In the “The Parachute Jump,” students work in pairs to design parachutes and use the Pythagorean theorem to calculate the distance between their “jumps.” Using the song “Ode to Pythagoras” (page 37) and their active involvement, students begin to see the real-world applications of algebra while using a variety of learning styles to better understanding this concept.

The Pythagorean theorem is revisited in “The Irrational Spiral” (page 43), but this time students experiment with algebra, geometry, and design. The lesson begins by “replaying” the musical ditty “Ode to Pythagoras.” Students, using protractors and rulers, draw a right triangle with a base and height of 1 cm and progress to design an amazing spiral design.

The activity “Seeing to the Horizon” (page 44) explains the scientific principle that on a clear day, “the higher up you are, the further you can see.” A list of the ten tallest skyscrapers is provided and, using an algebra formula, students calculate how far they can see from the rooftops of these buildings. This is the perfect time to work with the art teacher and introduce perspective and vanishing points to students.

Students continue to research the height of their school building, but this time using stopwatches and marshmallows. In “Falling Objects: Formulas in the Real World” (page 49), students start out with skyscrapers and then use marshmallows, stopwatches, and algebraic formulas to estimate the height of their school in a unique way.

And finally, “Pattern Block Patterns” (page 55) uses hands-on manipulatives to have students build geometric patterns. By describing, analyzing, and replicating the pattern, they are using algebraic reasoning while utilizing different learning styles and intelligences.

Cokie Roberts is quoted as saying, “As long as algebra is taught in school, there will be prayer in school.” Perhaps if students are given the opportunity to experience hands-on activities related to algebra, the subject will not generate such aversion and apprehension.

The Parachute Jump

MATH TOPICS

ordered pairs on the coordinate plane, the Pythagorean theorem, data collection and analysis

PRIOR KNOWLEDGE NEEDED

1. understanding of (x, y) coordinates on the coordinate plane
2. knowledge of the Pythagorean theorem

DIFFERENTIATION STRATEGIES

Principles

Flexible grouping: For this activity, students work in pairs. While this activity is designed to help students calculate the distance between two points on the coordinate plane using the Pythagorean theorem, the activity can be tiered. More advanced students can be taught the distance formula and use the formula to find the distance. $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Ongoing assessment: Observe students during the activity, and use data collection sheets to assess level of understanding.

Teacher's Strategies

Product: Tiered Journal Questions: The Level 1 question is an easier question because it asks students to explain, in their own words, what happened in the experiment that uses the Pythagorean theorem. The Level 2 question is written for students who used the distance formula. It asks them to explain why the distance formula is a shortened version of the Pythagorean theorem.

According to Students

Learning Styles/Multiple Intelligences: logical/mathematical, bodily/kinesthetic, musical/rhythmic, visual/spatial, verbal/linguistic, interpersonal

MATERIALS NEEDED

Each pair of students will need the following:

1. two coffee filters



2. eight 12-inch lengths of thread
3. a paper punch
4. large safety pins, large paper clips, or washers
5. colored pencils or markers
6. poster board or a large sheet of graph paper
7. a student activity sheet
8. data collection table

TEACHING SUGGESTIONS

Engaging the Students

Discuss with students the parachute jump experiment. Have two parachutes made to represent the two students' parachutes. Drop one previously made parachute onto a coordinate plane and mark that point. Now drop the second one; mark that point as well. Lightly draw a line connecting the two drop points. Discuss with students that the length and width of each of the squares on the graph represent a side of 1 and ask, "Can the distance between the two drop points be measured as if it were the side of a square?" Using a pencil, lightly draw a vertical line from one drop point and a vertical line from the other drop point. These two lines will form the legs of a right triangle—the line connecting the distance between the two drop points is the hypotenuse of this triangle. Discuss with students how the Pythagorean theorem can be used to approximate the length of the distance between the drop points.

The Exploration

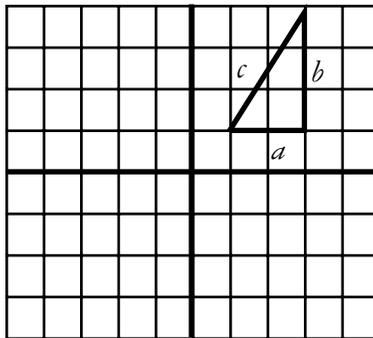
The song "Ode to Pythagoras" can be sung before this experiment is conducted. This is an interesting way to review the Pythagorean theorem and is most effective with students with a variety of learning styles.

Students are placed into pairs and given one student activity sheet. This sheet contains a set of directions to help them design and build their own parachute to use in the experiment and fill out the data collection table. Four holes are evenly spaced around the edge of a coffee filter, and string (about 12 inches long) is attached to each hole. A large paper clip or washer must be attached to the string to help the parachute land. Students may decorate their parachutes for purposes of identification.

Each team must divide a poster board into 4 quadrants, labeling the x and y coordinate points unless large sheets of graph paper are available for their use. (An overhead transparency can serve as a sample for the groups.)

Each team member, taking turns, drops his or her parachute onto the poster board. Each landing location (the x and y coordinate point) is recorded in the data collection table.

It is possible to form a right triangle by connecting the two landing points (this line becomes the hypotenuse of the right triangle) and then drawing the horizontal and vertical lines from these points to form the legs of the triangle. The lengths of the vertical line (side b) and horizontal line (side a) can be easily calculated and the Pythagorean theorem ($a^2 + b^2 = c^2$) can be used to find the distance between the two landing points. For example, if the two parachutes land on the coordinates indicated on this graph, the length of side $a = 2$ and the length of side $b = 3$; $2^2 + 3^2 = c^2$. $4 + 9 = c^2$; $c = \sqrt{13} \approx 3.6$. For those students using the distance formula, this formula can be easily substituted in the data collection table in the last column. Students will conduct ten drops and find the average or mean distance between them.



Debriefing

Give each group time to discuss their results and the mean distance between the ten drops. You can award a prize to the team with the shortest mean distance.

ASSESSMENT

1. Student products: Make sure students have successfully completed activity sheets.
2. Journal questions:
 - a. Level 1 question: Discuss the interesting aspects of your experiment by describing 1) how you used the Pythagorean theorem to help you find the distances, 2) why you could not just measure the distance between the two coordinate points, 3) your results (if landings in one quadrant occurred more often than another, if the parachute drifted or always landed in the same location, and why you think this occurred).
 - b. Level 2 question: Analyze both the Pythagorean theorem and the distance formula. Explain why the distance formula is actually the Pythagorean theorem expressed in another way and why either one can be used to find the hypotenuse of a right triangle.

Ode to Pythagoras

SUNG TO “HOKEY POKEY”

You take your first leg “*a*,”
And then your next leg “*b*,”
Take the sum of their squares,
Are you following me?

To use this famous theorem,
For Pythagoras let’s shout,
That’s what it’s all about.

We’re not quite finished yet,
Cause there’s a third side to see,
You know this tri-angles right,
Are you following me?

To use this famous theorem,
For Pythagoras let’s shout,
That’s what it’s all about.

Now add the square of leg “*a*,”
To the square of leg “*b*,”
You get the square of side “*c*,”
Are you following me?

To use this famous theorem,
For Pythagoras let’s shout,
That’s what it’s all about.

$$a^2 + b^2 = c^2$$

© 1998 Hope Martin

The Parachute Jump

Directions: Work with a partner to complete the following activity.

- A. You will need:
1. one coffee filter
 2. four pieces of string \approx 12 inches long
 3. a large paper clip, large safety pin, or washer
 4. one piece of poster board or large sheet of graph paper for you and your partner
 5. one data collection table for you and your partner
 6. colored markers or colored pencils
- B. You will need to create and design your own parachute. Punch four holes near the edge of a coffee filter at evenly spaced intervals. Attach four pieces of string (12 inches long) in each of the holes. Draw the four pieces of string together and attach a large paper clip, a safety pin, or a washer at the end of the connected string. Be sure to decorate your parachute for ease in identification.
- C. Work with your partner to create a “landing area” for the parachutes.
1. Divide the poster board or your large sheet of graph paper into 4 quadrants.
 2. Label the x and y coordinate points.
- D. Drop your parachutes (one at a time) from the designated height above the landing area. The “landing location” is the nearest (x, y) point on the “landing area.” Record ten successive drops for you and your partner on the data collection table.
- E. Use graph paper after each “drop” to plot the coordinate points for your drop and your partner’s drop. Connect the two points, and draw a horizontal line from one of the points and a vertical line from the other. You have now formed a right triangle. Use the Pythagorean theorem to find the length of the hypotenuse of this triangle.
- F. After your ten trials, find the mean difference of distance between the parachute drops. Compare your results with other groups. How did your distances compare?

The Parachute Jump

DATA COLLECTION

Trial Number	(x, y) Coordinates of Team Member 1	(x, y) Coordinates of Team Member 2	Length of Side a	Length of Side b	Distance Between Landing Locations of Parachutes 1 and 2: $a^2 + b^2 = c^2$
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
Mean					

Selected Answers

Page 11—All About the Moon

1. The crater is just slightly smaller than the size of South Carolina (31,000-sq. mi.) but more than three times the size of the state of New Jersey (7,787 sq. mi.).
2. It would take 8 years, 267 days for the train to reach the moon.
3. The moon rock costs \$30,000,000/lb. or \$1,875,000/oz.

Page 20—Baking Blueberry Muffins

- A.
1. 96 muffins
 2. \$0.375 each or $37\frac{1}{2}$ ¢ each
 3. 300% profit
- B.
1. 57 muffins (there is a remainder but that is not enough to make another muffin)
 2. \$0.64 each (although the remainder is less than $\frac{1}{2}$, it is added on to calculate cost)
 3. \$30.72/dozen (students may round up the number to a more reasonable amount)

Page 29—Diagramming Divisibility

The answers are located on page 27 of the teacher's page.

Page 47—Seeing to the Horizon

The answers are located on page 45 of the teacher's page.

Page 53—Falling Objects: Formulas in the Real World

The answers are located on page 51 of the teacher's page.

Page 57—Pattern Block Patterns

The fifth term will have 4 triangles and 5 trapezoids.

Since each term has $t-1$ # of triangles and t # of trapezoids, the 20th term will have 19 triangles and 20 trapezoids; the hundredth term will have 99 triangles and 100 trapezoids; the n th term will have $n-1$ triangles and n trapezoids.

Page 65—Pennies and the Sears Tower

The number of pennies needed to reach a height of 1450 feet will depend on the student-explorations. The students' calculations will depend on the age of their pennies, the number of pennies they used to make their estimates, and the accuracy of their measurements. Because of all of these variables, it is necessary to collect the data from all of the groups and analyze the individual results.

Page 106—Vowels, Vowels, Everywhere

The vowel A appears in the poem 41 times or 6.4% of the total # of letters (639).

The vowel E appears in the poem 95 times or 14.9% of the total # of letters (639).

The vowel I appears in the poem 55 times or 8.6% of the total # of letters (639).

The vowel O appears in the poem 43 times or 6.7% of the total # of letters (639).

The vowel U appears in the poem 16 times or 2.5% of the total # of letters (639).

Page 106—Vowels, Vowels, Everywhere

Vowels	Frequency	Fraction	Percentage	# of ° in circle
A	41	41/250	16.4%	59°
E	95	95/250	38.0%	137°
I	55	55/250	22.0%	79°
O	43	43/250	17.2%	62°
U	16	16/250	6.4%	23°
TOTAL	250	250/250 = 1	100.0%	360°

Page 120–121—Geometric Probability: Dartboards and Spinners

Answers are on page 118 of the teacher's page.

Based on the theoretical probabilities of each of the colors, the sections of the circle graph should be Red–60°, Yellow–120°, Green–100°, and Blue–80°.

Page 128—Flipping Three Coins: Heads or Tails

The theoretical probability of each occurrence is found on page 126 of the teacher's page.