

Active Learning in the Mathematics Classroom, Grades 5–8

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Preface

Education is not the filling of a pail, but the lighting of a fire.

—William Butler Yeats

An old Chinese proverb says, “I hear, and I forget; I see, and I remember; I do, and I understand.” Consider trying to learn to dance by reading a book and memorizing the steps or learning to drive a car by reading the manual . . . we learn when we are actively involved in the learning process and use a variety of learning modalities. Not all students have the same talents, learn the same way, or have the same interests and abilities. But all students must have access to high-quality mathematics instruction.

The National Research Council (1989) stated, “Research on learning shows that most students cannot learn mathematics effectively by only listening and imitating; yet most teachers teach mathematics this way. Most teachers teach as they were taught, not as they were taught to teach” (p. 6).

The Foundation Coalition (n.d.) asks “Why don’t we teach the way students learn?” Their *Cone of Learning* compares how much we remember with how it is taught. They maintain that we remember 10% of what we read, 20% of what we hear, 50% of what we both see and hear, but 90% of what we do—what we are actively involved in. By participating in a real experience or simulation, students are involved and motivated, and the learning curve rises dramatically. Ahmed (1987), in *Better Mathematics*, agrees: “Mathematics can be effectively learned only by involving pupils in experimenting, questioning, reflecting, discovering, inventing, and discussing” (p. 24).

Van de Walle (2006) describes the current teaching of mathematics in the following way: “Traditional teaching, still the predominant instructional pattern, typically begins with an explanation of whatever idea is on the current page of the text, followed by showing children how to do the assigned exercises. . . . The focus of the lesson is primarily on getting answers” (p. 12). The result of this style of instruction is that a large number of today’s students are not prepared for the realities of living in the 21st century.

The National Council of Teachers of Mathematics (NCTM; 2000) in *Principles and Standards for School Mathematics*, says, “In this changing world, those who understand and do mathematics will have significantly enhanced opportunities and options for shaping their future. A lack of mathematical competence keeps those doors closed.” The Council has called for “a common

foundation of mathematics to be learned by all students” (p. 5). In this document, the NCTM has divided 10 standards into two groups: the *Content Standards* and the *Process Standards*. The five Content Standards—number and operations, algebra, geometry, measurement, and data analysis and probability—clearly describe the content that students should learn. The five Process Standards—problem solving, reasoning and proof, communication, connections, and representation—illustrate what students should be doing to acquire and use the content knowledge. “This set of ten Standards does not neatly separate the school mathematics curriculum into nonintersecting subsets. . . . Process can be learned within the Content Standards and content can be learned within the Process Standards. Rich connections and intersections abound” (pp. 30–31).

The investigations in *Active Learning in the Mathematics Classroom* have been designed to keep students in Grades 5 through 8 *actively involved in the learning of mathematics*. I concur that “mathematics is not a spectator sport,” and this maxim has been the guiding principle behind the structure of this book. Chapters have been organized around the five Content Standards, but as stated above, the strands cannot be separated into nonintersecting subsets, and so all of the activities connect many mathematics skills and concepts. The way each activity ties to the standards is shown in Table P.1, Alignment With NCTM Standards, which immediately precedes Chapter 1.

This second edition has undergone some significant changes:

- The material has been updated and now includes Web sites that can be used as enrichment and bring technology into the mathematics classroom.
- A new chapter, “Active Algebra,” has been added to reflect the changes in the middle school mathematics curriculum.
- There are 15 new activities.
- The outline and organization of the chapters have been altered to better reflect the recommendations of the NCTM (2000) in the *Principles and Standards for School Mathematics* and current research on active learning.

These are brief descriptions of the chapters included in the second edition:

Chapter 1, “Investigations: Estimation, Large Numbers, and Numeration,” gives students the opportunity to collect data using very large numbers, use unique ways to revisit number theory, and explore real-world applications or recipes and music to reinforce their understanding of fractions.

Chapter 2, “Active Algebra,” is a new addition to the book. Ten years ago it was not common for middle school students to be studying algebra. But in today’s classrooms, many students of this age are introduced to or actively involved in the study of this strand of mathematics. For this reason, motivating activities have been added to meet these needs. Manipulatives are used to give students a tactile experience with patterns; algebra jokes provide a self-correcting way to help students solve linear equations and solve geometric problems; data collection and the use of variables are merged in a tasty activity; and verbal, algebraic, and graphic representations are combined to help students make important connections.

Chapter 3, “Geometry: Our Mathematical Window to the World,” includes a variety of activities to help students progress through van Hiele’s levels of geometric learning (as cited in Fuys, Geddes, & Tischler, 1988). Dina van

Table P.2 van Hiele's First Three Levels

<i>Level</i>	<i>Name</i>	<i>How Might the Child Describe a Parallelogram?</i>
0	Visual level/Visualization	It looks like a slanted rectangle.
1	Descriptive level/Analysis	It has parallel sides.
2	Informal deductive level/ Reasoning	It is a quadrilateral whose opposite sides are parallel and equal.

Hiele-Geldof and Pierre van Hiele hypothesized five levels of geometric thinking. They believed that only when children achieved a firm understanding of geometric concepts at a lower level were they able to move on to a higher level. While the van Hieles put forward five levels, only three of these strongly relate to students at the middle school level. These are shown in Table P.2.

If a child is still at Level 0, “It looks like it, therefore it is,” the van Hieles contend that they cannot move on to Level 1, the descriptive level—a higher level of understanding. For this reason, hands-on activities, such as paper folding, measuring, and building, help students discover empirically the properties or rules of a class of both two-dimensional and three-dimensional shapes.

Chapter 4, “The Measure of Mathematics,” employs current research recommendations by getting students actively involved in tasks that use both metric and customary units, using scale factors and selecting appropriate units of measure. Students are encouraged to measure and then use ratio, proportion, and scale drawing to reproduce the Statue of Liberty, make use of the distance formula to walk through the center of the Earth, experiment with scatterplots as a way to compare two measurements, and make real-world consumer connections.

Chapter 5, “Data Collection and Probability,” demonstrates the data collection process by taking students through the four steps: collection, organization, analysis, and graphic representation. In today's world, we are overwhelmed with a plethora of statistics. Using data supplied by the National Basketball Association (NBA), students are encouraged to recognize the value of graphic representations to make sense of a large amount of data. The three activities related to probability ask students to discover possible outcomes, determine the “fairness” of a game based upon odds, and make predictions based upon their experimental results.

HOW THE CHAPTERS ARE ORGANIZED

Brief Introduction: At the beginning of each chapter, there is a brief description of the activities and the mathematical concepts contained in the chapter.

Teacher's Pages: There are several pages of planning information for each activity. These pages contain the following:

1. *Math Topics:* This is a listing of the mathematics skills and concepts that are contained in this lesson. Because of the richness of the activities, there will always be more than one skill listed in this area.

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2. *Active Learning:* This section lists what the students will do. It may also recommend a flexible grouping arrangement. A careful analysis of these concepts will help define the components for authentic assessment.

3. *Materials:* This section lists the supplies and materials needed for students to begin the activity. For example, if an overhead transparency is needed for follow-up work with students, it will be listed here. If calculators are listed, they are necessary because of the complexity of the computations. Having these supplies and materials on hand will help make the activity run smoothly and efficiently.

4. *Suggestions for Instruction:* This area has been expanded in this edition. It has been designed to give the teacher an idea of how to begin and of what might happen during the lesson. Most often, this section begins with a question to be posed to the students. For example, “What do you think will happen if . . . ?” “Have you ever considered the possibility that . . . ?” or “How do you think we might attempt to figure that out?” You would never want to say something like, “Don’t do it that way” or “You must always do it this way.” Instead, serve as a facilitator or guide to the students. I would caution you to follow two very important rules:

- Go with the flow—give your students the opportunity to explore their own solutions. It is amazing the creativity students will use when they can plan and organize in a cooperative group.
- Don’t be the sage on the stage—don’t feel that you must tell your students “how to do it” or what the answer is. Give them the opportunity to experiment with various strategies and use their multiple intelligences and learning strengths. When you give students a “rule,” you are limiting their solution strategies to one right way and may, in fact, be hindering rather than encouraging their learning.

If there are interesting Web sites or references that can be used to enhance the lesson, these will be included in this section as well.

5. *Selected Answers:* When a lesson has a unique answer, it will be found in this section, rather than at the back of the book as in the first edition.

6. *Variation:* This section describes supplementary or extension activities for the lesson. It might extend the activity to a longer project, develop the same concepts using additional activities, or be something that challenges your “better” math students. Use your creativity—add variations and design your own activities using the materials and models.

7. *Writing in Math:* The journal questions provide students an opportunity to express their knowledge using language rather than mathematical symbols and rules. In this edition, it might include a problem similar to the one in the activity and so it can be used as one assessment component. Or it might ask students to describe the procedures they used while solving the problem. Expressing their thoughts in this way gives the teacher a clearer indication of how well they each understood the mathematics embedded in the activity. In this edition, there will usually be two journal questions. These can be used to

differentiate instruction, as one of the questions is usually more difficult than the other.

The Math Investigation: ready-to-use activities follow the teacher's planning pages. The 15 new activities in the second edition include investigations related to algebra, data collection, and measurement. In addition, many of the activities have been revised and updated. Interesting Web sites have been added to provide technology options. All the activities have all been designed to give students the opportunity to become actively involved in learning mathematics by using manipulatives, making connections to real-world applications, or addressing their different learning styles and multiple intelligences.

At the back of the book, there is a Resource section that lists interactive Web sites to make math–technology connections. Following it, the Bibliography consists of a list of references related to curriculum, assessment, and current research. The Web sites listed are those that encourage student's active involvement. Many of these sites can be used to enrich topics other than those presented in this book. Be sure to explore them fully and add them to your activity list.

What we learn to do, we learn by doing.

—Aristotle